Kuwait University	Math 101	Date:	November 16, 2009
Dept. of Math. & Comp. Sci.	First Exam	Duration:	90 minutes

Calculators, cellular phones and all other mobile communication equipments are not allowed.

Answer the following questions:

1. Use the definition of the derivative to find $f'(2)$, where

$$f(x) = \sqrt{x+2}.$$

(3 pts.)

2. Find $f'(x)$, where

$$f(x) = \sin^3(x^2+7).$$

(3 pts.)

3. Evaluate the following limit, if it exists

$$\lim_{x \to -2} \frac{\sin(x+2)}{x^3+8}.$$

(3 pts.)

4. Find the vertical and horizontal asymptotes, if any, for the graph of the function

$$f(x) = \frac{x-1}{|x+1|}.$$

(3 pts.)

5. (a) State the Intermediate Value Theorem. (1pt.)

   (b) Show that the equation

$$2x^5 + 2x^2 + x - 12 = 0$$

   has a real solution. (3 pts.)

6. Let $f(x) = \dfrac{x^{\frac{1}{3}}}{x-2}$. Find the x-coordinate(s) of the point(s) on the graph of f at which

   a) the tangent line is horizontal

   b) the tangent line is vertical. (4 pts.)

7. Find the x-coordinates of the points at which the function f is discontinuous, where

$$f(x) = \begin{cases} \dfrac{1-\cos \pi x}{x} & \text{if } x < -1, \\ x^2 \cos \frac{\pi}{x} & \text{if } -1 \le x < 0, \\ \dfrac{\tan 5x}{x} - 5 & \text{if } x > 0. \end{cases}$$

   Classify the types of discontinuity of f as removable, jump, or infinite. (5 pts.)

Kuwait University	Math 101	Date: July 18, 2009
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1. $f'(2) = \lim\limits_{x\to 2} \frac{\sqrt{x+2}-2}{x-2} \times \frac{\sqrt{x+2}+2}{\sqrt{x+2}+2} = \lim\limits_{x\to 2} \frac{x+2-4}{(x-2)\left(\sqrt{x+2}+2\right)} = \lim\limits_{x\to 2} \frac{1}{\sqrt{x+2}+2} = \boxed{\frac{1}{4}}$.

2. $f'(x) = \boxed{6x\sin^2(x^2+7)\cos(x^2+7)}$.

3. $\lim\limits_{x\to -2} \dfrac{\sin(x+2)}{x^3+8} = \lim\limits_{x\to -2} \dfrac{\sin(x+2)}{(x+2)(x^2-2x+4)} = \lim\limits_{x\to -2} \dfrac{\sin(x+2)}{(x+2)} \times \lim\limits_{x\to -2} \dfrac{1}{(x^2-2x+4)} = 1 \times \frac{1}{(4+4+4)} = \boxed{\frac{1}{12}}$.

4. $\lim\limits_{x\to -1^{\pm}} \dfrac{x-1}{|x+1|} = \lim\limits_{x\to -1^{\pm}} \frac{x-1}{\pm(x+1)} = \boxed{-\infty} \Longrightarrow \boxed{x=-1}$ is V.A.

   $\lim\limits_{x\to \pm\infty} \dfrac{x-1}{|x+1|} = \lim\limits_{x\to \pm\infty} \dfrac{x-1}{\pm(x+1)} = \lim\limits_{x\to \pm\infty} \dfrac{x\left(1-\frac{1}{x}\right)}{\pm x\left(1+\frac{1}{x}\right)} = \boxed{\pm 1} \Longrightarrow \boxed{y=1 \ \& \ y=-1}$ are H.A.

5. (b) Let $f(x) = 2x^5 + 2x^2 + x - 12$. f is continuous on $[0,2]$ (f is a polynomial function). Since $f(0) = -12 < 0$ and $f(2) = 62 > 0$, by IVT f has at least one real root in $(0,2)$.

6. $f'(x) = \frac{-2-2x}{3x^{\frac{2}{3}}(x-2)^2}$
   a) $f'(x) = 0$ at $x=-1$ $\therefore$ f has a *horizontal tangent* at $\boxed{x=-1}$.
   b) f is continuous at $x=0$ and $\lim\limits_{x\to 0^{\pm}} f'(x) = \boxed{-\infty}$ $\therefore$ f has a *vertical tangent* at $\boxed{x=0}$.
   Note that $2 \notin D_f$.

7. f is discontinuous at $x = 0, -1, \frac{\pi}{10} + \frac{k\pi}{5}$.
   For $\boxed{x=0}$: f is not defined at $x=0$.
   $\lim\limits_{x\to 0^+} \left[\dfrac{\tan 5x}{x} - 5\right] = 5 \lim\limits_{x\to 0^+} \dfrac{\tan 5x}{5x} - 5 = 5(1) - 5 = \boxed{0}$.
   For $\lim\limits_{x\to 0^-} x^2\cos\frac{\pi}{x} = ?$. We know that $-1 \leq \cos\frac{\pi}{x} \leq 1$, thus $-x^2 \leq x^2\cos\frac{\pi}{x} \leq x^2$.
   Since $\lim\limits_{x\to 0^-} -x^2 = 0 = \lim\limits_{x\to 0^-} x^2$, by the Squeeze Theorem $\lim\limits_{x\to 0^-} x^2\cos\frac{\pi}{x} = \boxed{0}$.
   $\therefore$ f has *a removable discontinuity* at $x=0$.

   For $\boxed{x=-1}$:
   $\lim\limits_{x\to -1^+} x^2\cos\frac{\pi}{x} = (-1)^2\cos\frac{\pi}{-1} = \boxed{-1}$.
   $\lim\limits_{x\to -1^-} \dfrac{1-\cos(\pi x)}{x} = \dfrac{1-\cos(\pi(-1))}{-1} = \boxed{-2}$.
   $\therefore$ f has *a jump discontinuity* at $x=-1$.

   For $\boxed{x = \frac{\pi}{10} + \frac{k\pi}{5}}$, $\forall k \in N$, f has *infinite discontinuities.*